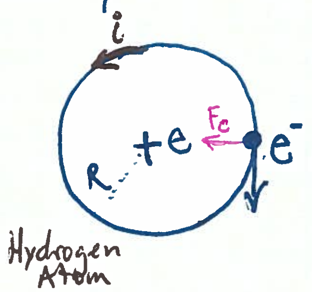
8.02x w/ Lewin | LECTURE 22 | 4/7/2020

Effect of magnetic field on materials, historicis

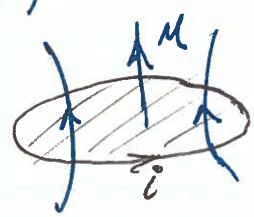
Last time we saw...
When we expose a material to an exterior magnetic field then the field inside the material is modified.

$$\vec{B} = k_m \vec{B}_{external}$$

Now...
Calculate Magnetic Dipole Moment of an atom (μ)
(need quantum mechanics... not done here)



$M_e = 9.1 \times 10^{-31}$ kg
$e = 1.6 \times 10^{-19}$ C
$R = 5 \times 10^{-11}$ m



$\boxed{\mu = IA}$

$A = \pi R^2 \simeq 8 \times 10^{-12}\ m^2$

Coulomb: $F = \frac{ee}{4\pi\epsilon_0 R^2}$

Centripedal Force: $F = \frac{mv^2}{R}$

Velocity of electron

$$\frac{e^2}{4\pi\epsilon_0 R^2} = \frac{mv^2}{R}$$

$$\boxed{v = \sqrt{\frac{e^2}{m 4\pi\epsilon_0 R}}}$$

$V_{electron} \simeq 2.3$ km/s

time for electron to go around circle: $\boxed{T = \frac{2\pi R}{v}} \simeq 1.4 \times 10^{-16}$ sec

Current: $\boxed{I = \frac{e}{T}} \simeq 1.1 \times 10^{-3}$ A

Magnetic Moment: $\boxed{\mu = IA} = 9.3 \times 10^{-24}\ Am^2$ Bohr Magneton

How strong of a field can we create if we align all magnetic dipoles?

Aligning dipoles

$$\vec{B} = \vec{B}_{vac} + \vec{B}'$$

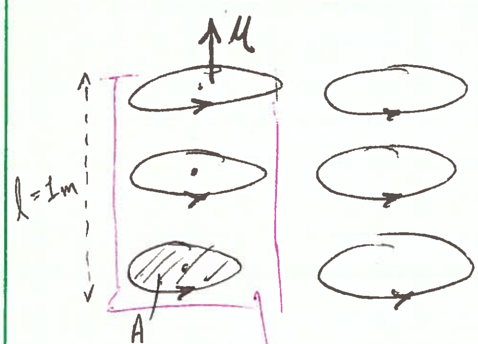
if $B' \propto B_{vac}$

then $\vec{B}' = \chi_m \vec{B}_{vac}$

So $\boxed{\vec{B} = (1 + \chi_m)\vec{B}_{vac}}$

$B' \propto B_{vac}$ for paramagnetic materials but ferromagnetic materials do not always have this (bc ferromag. already well aligned mag. dipoles in domains)

Choose material with $\mu = 2\mu_{bohr}$, $\mathcal{N} = 10^{29}$ atoms/m^3



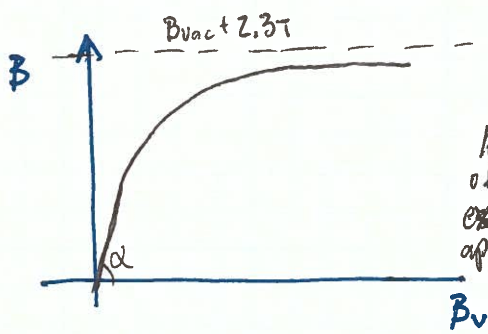
for a solenoid: $B = \frac{\mu_0 I N}{L}$ remember

Choose $\mathcal{N}$ atoms : 10^{29} atoms/m^3
(number density)

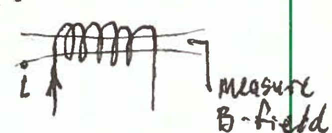
atoms per meter : $A\mathcal{N}$
(# turns per meter) $\rightarrow N/l$

So... $\boxed{B = \mu_0 I A \mathcal{N}}$ $\approx 2.3\,T$

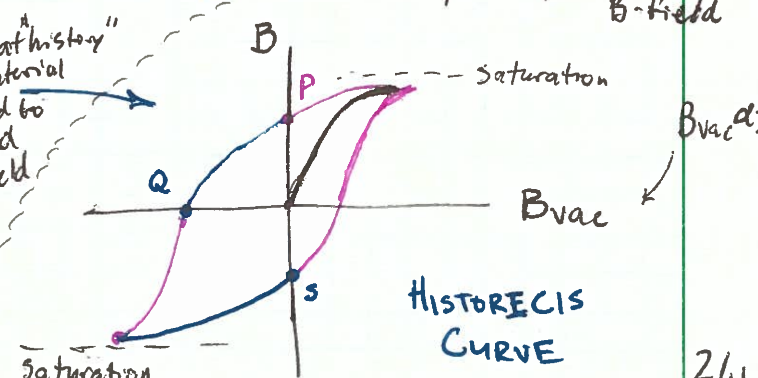
Put ferromagnetic material in external B-field (solenoid). plot applied field vs. current in solenoid



$B_{vac} = \mu_0 I \frac{N}{l}$



"look at history" of material exposed to applied field



HISTORECIS CURVE

At P: $B_{vac} = 0$, $B' \rightarrow$
Q: $B_{vac} \leftarrow$, $B' = 0$

Historicis Demo

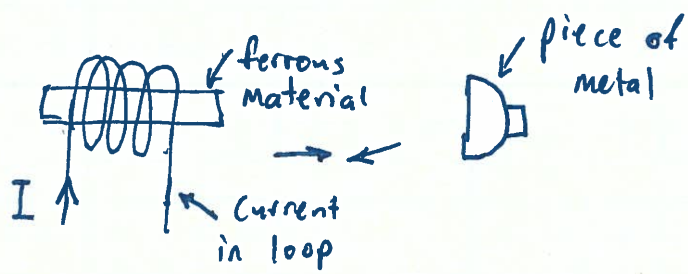
Apply 60 Hz AC through solenoid to generate applied field B_{vac}, insert ferrous material and measure total field B

Demagnetize Magnet By:

$$B_{vac} = \mu_0 I \frac{N}{\ell}$$

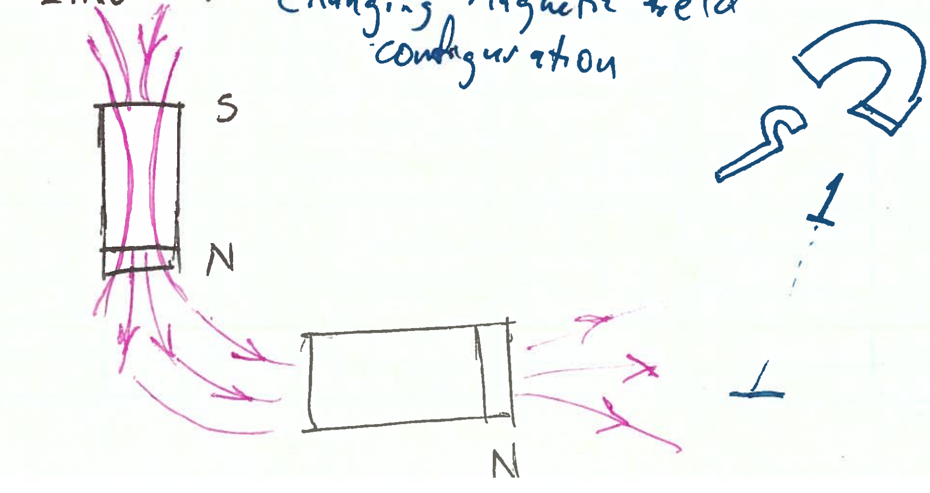
1. heating it up
2. Banging on it
3. Reduce B_{vac} (here w/ current in solenoid I can just reduce current because $B \propto I$)

Electromagnet Demo



Even when we remove current from loop, it is still a magnet! This is historicis!!! Domains still aligned!

Demo: Changing Magnetic field configuration



Nail attracted to permanent magnet but when iron wrench introduced the field is altered and nail drops down

Now let's add effect of magnetic fields to Maxwell's Eqns

Gauss $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{K\varepsilon_0}$ ✓ OK

Gauss $\oint \vec{B}\cdot d\vec{A} = 0$ ✓ OK

Faraday $\oint E\cdot dl = -\frac{d\Phi_B}{dt}$ ✓ OK

Ampère $\oint B\cdot dl = K_m \mu_0\left(I_{pen} + \varepsilon_0 K \frac{d}{dt}\Phi_E\right)$ must account for effect of magnetic historicis

MAXWELL'S EQUATIONS $\vec{D} = \varepsilon_0\vec{E}$, $\vec{H} = \mu_0\vec{B}$

$\nabla\cdot\vec{D} = \rho$ — Gauss — $\oint\!\!\oint_S \vec{D}\cdot d\vec{s} = \iiint_V \rho\, dV$

$\nabla\cdot\vec{B} = 0$ — Gauss — $\oint\!\!\oint \vec{B}\cdot d\vec{s} = 0$

$\nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B}$ — Faraday — $\oint_l \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\iint_S \vec{B}\cdot d\vec{s}$

$\nabla\times\vec{B} = \mu\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$ — Ampere — $\oint \vec{B}\cdot d\vec{l} = \mu_0\left(\iint_S \vec{J}\cdot d\vec{s} + \frac{d}{dt}\iint_S \vec{D}\cdot d\vec{s}\right)$