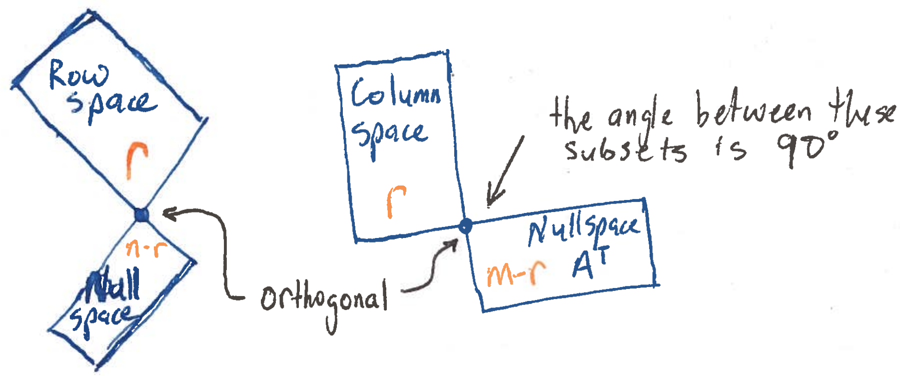
Lecture 14 — 12/3/2018

Orthogonal Vectors and Subspaces

Big Picture:



Orthogonal Vectors:

- Two vectors x,y are orthogonal if ~~their~~ dot product is zero
  ↳ $x^T y = 0$

- Pythagoras: $\|x\|^2 + \|y\|^2 = \|x+y\|^2$
  for right triangles!

$$\underline{x^T x} + \underline{y^T y} = (x+y)^T(x+y)$$

$$= \cancel{x^T x} + \cancel{y^T y} + x^T y + y^T x$$

$$x^T y + y^T x = 0$$

$$2x^T y = 0 \longrightarrow \underline{\underline{x^T y = 0}}$$

dot product from Pythag. Thm

Orthogonal Subspace

Subspace S is orthogonal to subspace T means every vector in S is orthog to every vector in T.

$\boxed{\text{Row space is orthogonal to null space of } A}$

$$Ax = 0 \rightarrow \begin{bmatrix} \text{row 1 of } A \\ \text{row 2} \\ \vdots \\ \text{row n of } A \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

↳ x is orthogonal to each row of A!

↳ x is ⊥ to all combinations of rows of A

$(\text{row } 1)^T x = 0$

$\vdots$

$\boxed{\text{column space is orthogonal to nullspace of } A^T}$

$$A^T x = 0 \rightarrow \begin{bmatrix} \text{col 1 of } A \\ \vdots \\ \text{col } m \text{ of } A \end{bmatrix} \begin{bmatrix} \\ x \\ \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \\ 0 \end{bmatrix}$$

FACT:

Nullspace and row space are orthogonal complements in R^n

$\longrightarrow$ nullspace contains all vectors $\perp$ to row space

$A^T A$ is invertible exactly if A has independent columns