

Digital Logic

PhD Qualifying Exam

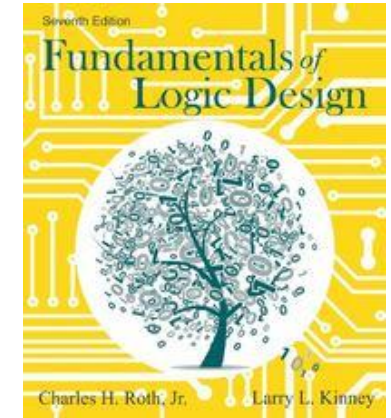
Review Session

November, 2017

Quincy FLINT

Resources

- Textbooks (*I don't think you need one*)
 - Fundamentals of Logic Design – Roth
- UF Past Practice Exams
 - <https://www.ece.ufl.edu/content/phd-written-qualifying-exam-questions>
- UF Exam Study Guide
 - <https://www.ece.ufl.edu/sites/default/files/pictures/DigitalLogic.pdf>
- Exam Registration
 - <https://gradadmissions.ece.ufl.edu/srs-servlet/examRegistration/phd>



Important Dates

- **12/01/2017** – Qualifying exam registration closes (Friday, 5 pm)
- **12/??/2017** – Potential 2nd review session
- **01/??/2018** – Final review session
- **01/20/2018** – Qualifying exam (Saturday, time TBD)

Boolean Algebra

- **Algebra with 0's and 1's**

- $X + 0 = X$

- $X + 1 = 1$

- $X * 1 = X$

- $X * 0 = 0$

- **Idempotent Laws**

- $X + X = X$

- $X * X = X$

- **Complement Laws**

- $X + /X = 1$

- $X * /X = 0$

Boolean Algebra

- **Dual:**

- $1 \rightarrow 0$
- $+ \rightarrow \times$

$$X + 0 = X \rightarrow X * 1 = X$$

$$X + 1 = 1 \rightarrow X * 0 = 0$$

$$X + /X = 1 \rightarrow X * /X = 0$$

- **DeMorgan's Laws**

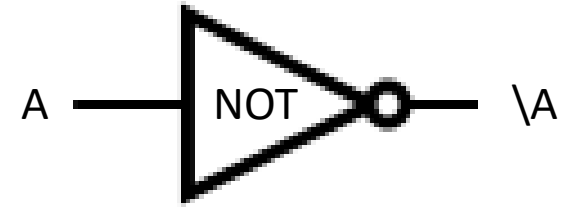
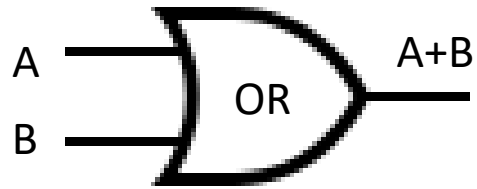
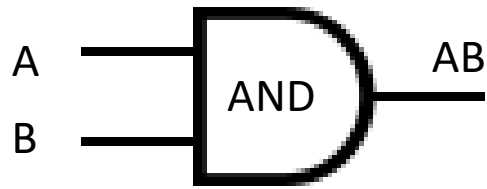
- $\text{NOT}(X+Y+Z) = \text{NOT}(X) * \text{NOT}(Y) * \text{NOT}(Z)$
- $\text{NOT}(X*Y*Z) = \text{NOT}(X) + \text{NOT}(Y) + \text{NOT}(Z)$

- **Consensus Theorem**

- $XY + YZ + \backslash XZ = XY + \backslash XZ$

Proof - example

Logic Gates



INPUT		OUTPUT		
A	B	A AND B	A OR B	NOT A
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

More Logic Gates

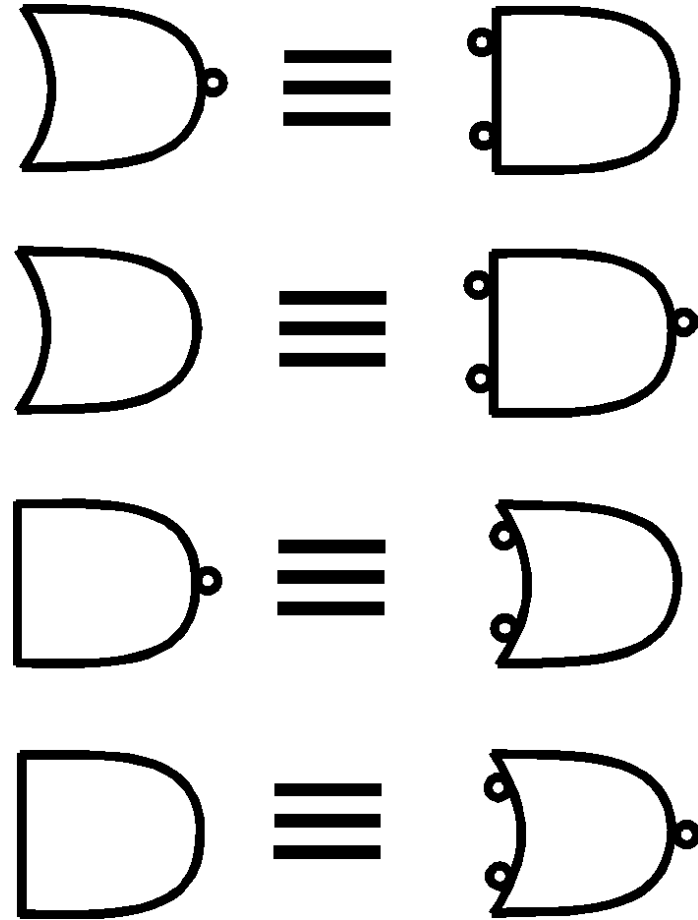
- **Active High**

- High Voltage (1) is TRUE

- **Active Low**

- Low Voltage (0) is TRUE

- Active Low to bubble = match
- Active High to bubble = mismatch
- Bubbles do not change logic



Number Systems

- **Base 10:** $541 = 5 \times 10^2 + 4 \times 10^1 + 1 \times 10^0$
 $= 5 \times 100 + 4 \times 10 + 1 \times 1$

- **Base 2:** $0101 = 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
 $= 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$

Conversion between systems

- **Decimal to Binary**

1. Brute Force -- “Count it out”
2. Divide by 2 – remainder becomes binary (least to most significant)

- **Decimal to Hex**

1. Convert to Binary then groups of 4 bits
2. Divide by 16 -- remainder becomes hex (least to most significant)

- **Decimal to Octal**

1. Convert to Binary then groups of 3 bits
2. Divide by 8 -- remainder becomes octal (least to most significant)

Signed Number Representations

- **Signed Magnitude:**
 - MSB gives sign
 - 1000 0101 = -5
- **1's Complement:**
 - if MSB is 1 – flip bits and apply minus sign
 - If MSB is 0 – do nothing, positive
 - 1111 1100 (flipped = 0000 0011) = -3
- **2's Complement:**
 - if MSB is 1 – flip bits, add 1, and apply minus sign
 - 1111 1101 (flipped + 1 = 0000 0011) = -3

Alternate 2's Complement Solution

- 2's Complement:

• **1011**

1. Flip Bits: 0100
2. Add 1: 0101
3. Interpret: -5

- 2's Complement:

• **1011**

$$\begin{array}{ccccccc} | & & | & | & & & \\ -8 & + & 2 & + & 1 & = & -5 \end{array}$$

Arithmetic

- Think back to basic arithmetic in base 10
- Let's just do some problems

SOP, POS, and K-Maps

- SOP
 - $XY + YZ$
- POS
 - $(X+Y)(Y+Z)$
- K-Map
 - Truth table to K-Map form
 - Equation to K-Map

		AB			
		00	01	11	10
CD	00				
	01				
	11				
	10				

Decoder

- Maps binary input to decimal output

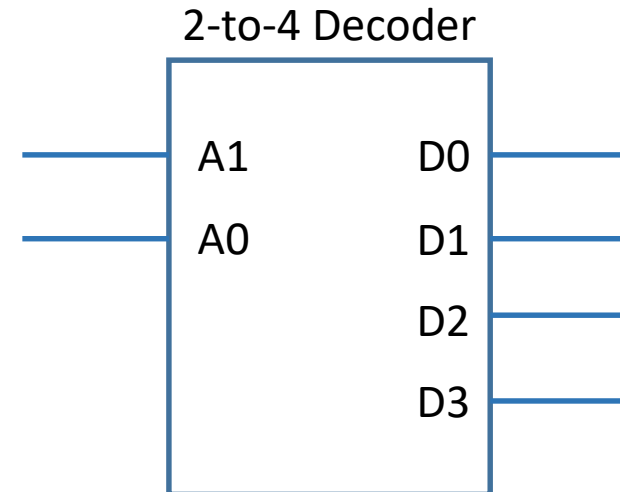
INPUTS		OUTPUTS			
A1	A0	D3	D2	D1	D0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

$$\neg A1 * \neg A0 \rightarrow D0$$

$$\neg A1 * A0 \rightarrow D1$$

$$A1 * \neg A0 \rightarrow D2$$

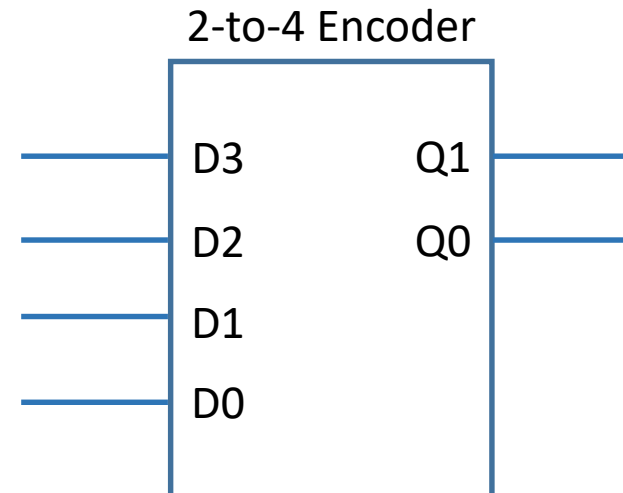
$$A1 * A0 \rightarrow D3$$



Encoder

- Maps decimal input to binary output

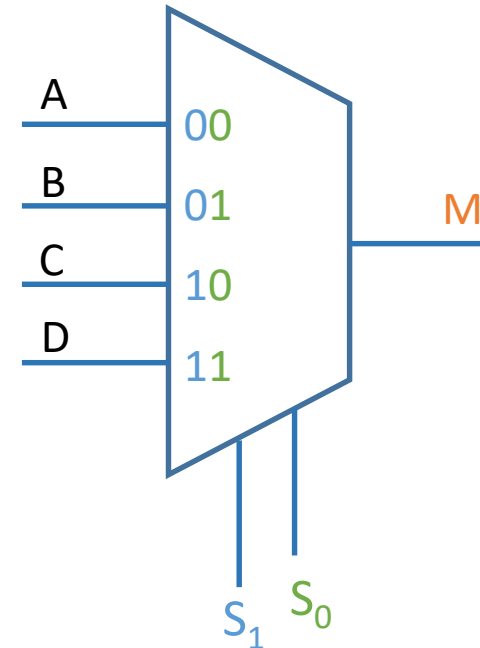
INPUTS				OUTPUTS	
D3	D2	D1	D0	Q1	Q0
0	0	0	0	X	X
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1



Mux

- Multiple input, single output selection device
 - $M = \overline{S_1} \text{ and } \overline{S_0} \text{ and } A$
+ $\overline{S_1} \text{ and } S_0 \text{ and } B$
+ $S_1 \text{ and } \overline{S_0} \text{ and } C$
+ $S_1 \text{ and } S_0 \text{ and } D$

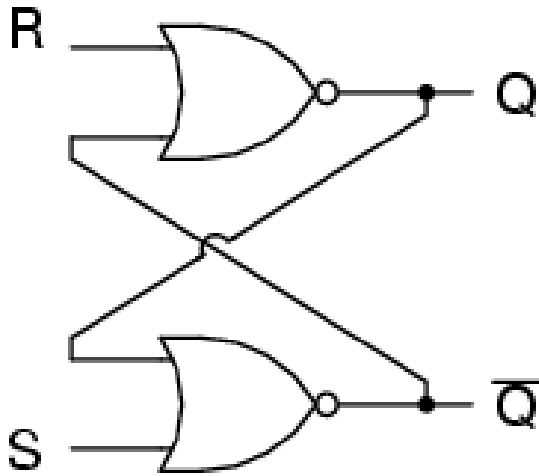
We can implement logic equations using multiplexers - example



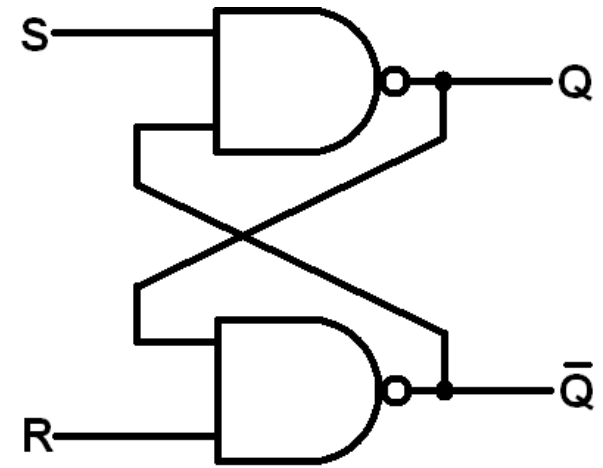
SR Latch

- NOR and NAND configurations

$$Q^+ = S + \bar{R}Q$$



S	R	Q	\bar{Q}
0	0	latch	latch
0	1	0	1
1	0	1	0
1	1	UND	UND



Flip-Flops

FLIP-FLOP NAME	FLIP-FLOP SYMBOL	CHARACTERISTIC TABLE	CHARACTERISTIC EQUATION	EXCITATION TABLE																																			
SR		<table border="1"> <thead> <tr> <th>S</th> <th>R</th> <th>$Q_{(next)}$</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> </tr> <tr> <td>0</td> <td>1</td> <td>0 Reset</td> </tr> <tr> <td>1</td> <td>0</td> <td>1 Set</td> </tr> <tr> <td>1</td> <td>1</td> <td>?</td> </tr> </tbody> </table>	S	R	$Q_{(next)}$	0	0	Q	0	1	0 Reset	1	0	1 Set	1	1	?	$Q_{(next)} = S + R'Q$ $SR = 0$	<table border="1"> <thead> <tr> <th>Q</th> <th>$Q_{(next)}$</th> <th>S</th> <th>R</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>0</td> <td>X</td> </tr> <tr> <td>0</td> <td>1</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>X</td> <td>0</td> </tr> </tbody> </table>	Q	$Q_{(next)}$	S	R	0	0	0	X	0	1	1	0	1	0	0	1	1	1	X	0
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Design a Counter Example

- **Example:** Count 7, 0, 3 using J-K Flip Flop and T-FF

State Machines

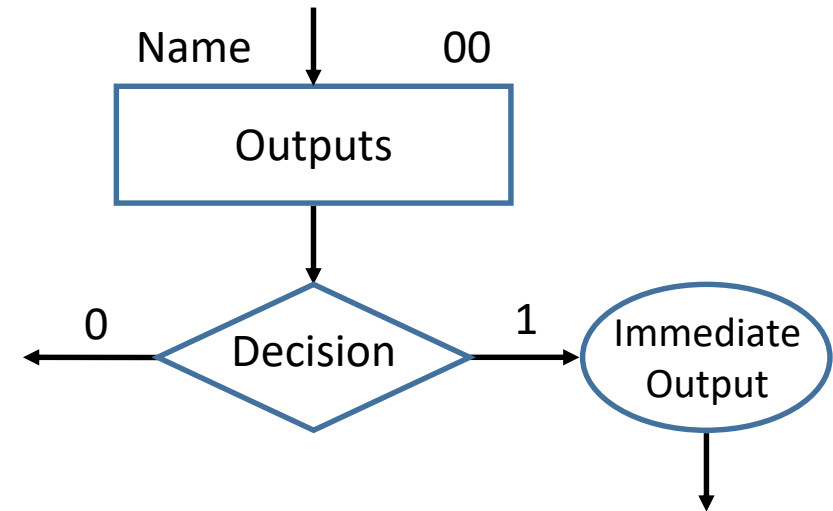
- Moore Machines
 - Output only depends on current state
- Mealy Machine
 - Output depends on current state and input
 - Asynchronous
- **Example:** Detect sequence 010^*1

ASM Diagrams

- Rectangle:
 - State box
 - Outputs go inside
- Diamond
 - Decision box, follows state box
 - Conditional Branch
 - Inputs go inside
- Oval:
 - Conditional (Mealy) outputs
 - Outputs go inside, asynchronous

- **Examples:**

- Detect sequence 010*1
- Washing Machine



Registers

- Just a collection of Flip-Flops
- **Example:**
 - Design 4-bit shift register with the following specs
 - Global enable
 - Asynchronous reset to 1010
 - We can give 4-bit input
 - Shift left and wrap
 - Shift right and sign extend